Effects of Surface Tension on Two-Dimensional Two-Phase Stratified Flows

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I. Introduction

Surface tension plays an important role in the evolution of the interface between two phases by smoothing sharp interfaces and minimizing the interfacial area. This in turn affects the flowfield of both phases. Therefore, an understanding of the effect of surface tension on low Reynolds number two-phase flow is essential.

One difficulty in simulating two-phase flows is due to the presence of an interface in the domain. Several methods are available to model the interface. These include but are not limited to the level-set (LSET) method, the volume-of-fluid (VOF) method, and the front-tracking method. In these methods, the effect of surface tension can be modeled using the continuum surface force (CSF) model as a concentrated force at the interface. Because the surface tension force is proportional to the interface curvature, accurate evaluation of the interface curvature is essential. Among the aforementioned methods, the LSET method offers a convenient way to calculate the curvature.

In this Note, the effect of surface tension on the interface evolution of a steady, two-dimensional, stratified two-phase flow is investigated using the LSET method. The surface tension force is modeled using the CSF approach. Localized mass correction (LMC) scheme is utilized to ensure mass conservation at every cross section.

II. Mathematical Formulation

A stratified flow of two immiscible fluids, as shown in Fig. 1, is considered. When a combined formulation is used, the continuity and momentum equations can be written as

$$\frac{\partial \rho u_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \rho \frac{\partial u_j}{\partial x_j} \right) - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_j}{\partial x_i} \right) + \rho g_i + D(\xi) \kappa \kappa_n$$

where $\xi$ is the distance function, $\kappa$ is the curvature, $\kappa_n$ is the normal curvature of the interface, $\rho$ is the density, $u_j$ is the velocity component in the $x_j$ direction, $p$ is the pressure, $\mu$ is the viscosity, $g_i$ is the gravity component in the $x_i$ direction, and $D(\xi)$ is the surface tension density.

The initial condition is

$$\xi(x, 0) = \xi_0(x)$$

and the boundary condition is

$$\frac{\partial \xi}{\partial x_j} = 0$$

To ensure that $\xi$ remains a distance function for all time, redistancing is performed. This is achieved by solving for the steady-state solution of a second distance function $\psi$ given by

$$\frac{\partial \psi}{\partial t} = \text{sign}(\xi) (1 - |\nabla \psi|)$$

The initial condition is $\psi(x, 0) = \xi_0(x)$. The LMC procedure is used to ensure mass conservation. The steady-state solution to a third distance function $\psi'$ is obtained using

$$\frac{\partial \psi'}{\partial t'} = \text{sign}(\xi_{\text{net}}) \frac{(\dot{m}_d - \dot{m}_s)}{\dot{m}_d}$$

where $\dot{m}_s$ and $\dot{m}_d$ are the current and desired mass flow rates, respectively. The steady-state values of the second distance function

![Fig. 1 Stratified two-phase flow between parallel plates.](image-url)
ψ are used as the initial condition for Eq. (7). Either fluid can be used as the reference fluid for the LMC procedure.

The velocity at the inlet is specified. No-slip condition is applied at the wall. At the outlet, zero gradient of the velocity is imposed. The LSET function is specified at the inlet. At the wall and the outlet, the zero gradient condition is used for the LSET function. Details of the solution procedure are given in Ref. 5 and will not be repeated here.

### III. Results and Discussion

Figure 1 shows the schematic of a stratified two-phase flow between two parallel plates. Two immiscible fluids flow a distance L between two parallel plates separated by a height W. At the inlet, the interface is located at δin. The development of the velocity profiles in the developing region causes the interface δ to evolve. Once the fully developed region is reached, the velocity profile and the interface location δin become independent of the axial coordinate.

For given ρ1/ρ2, μ1/μ2, V1/V2, and δin/W combination, three additional dimensionless parameters, namely, the Reynolds number Re = ρ2u0W/μ2, the capillary number Ca = u0μ2/σ, and the Eotvos number Eo = |μ1 − μ2|V2W/σ, are required to characterize the flow.

Here ρ1/ρ2, μ1/μ2, V1/V2, and Reynolds number Re are set to 2.0, 2.0, 1/8, and 0.01, respectively. V1/V2 is the volumetric flow rate ratio of the two fluids. Figure 2 shows the effects gravity and size (for a given |μ1 − μ2| and σ), presented in the form of Eo (δin/W = 0.3 and Ca = 0.1). As seen in Fig. 2, the interface evolution is not affected by gravity as long as Eo ≤ 1. As the Eotvos number Eo decreases with the channel size, the effects of gravity becomes less important in the microchannels. Also notice that the development length increases as the channel becomes smaller (smaller Eotvos number Eo). A maximum development length is observed. Once this length is reached, further decrease in Eotvos number Eo does not alter the development length. Because the solutions are invariant for Eo < 1, Eo is set to 0 for the remainder of this Note. As a result, the development lengths are the maximum possible lengths.

Figure 3a shows the evolutions of both the interface and the velocity field along the flow direction, up to x/W = 1, where changes are significant. The interface and the velocity profiles of the present LSET solution agree well with that of the VOF. Figure 3b shows the pressure distribution across the flow direction. For a comparison to be made, the pressure field at a given axial location is adjusted by adding a constant so that the pressure at the lower plate is zero. Therefore, the axial pressure gradient dp/dx cannot be inferred from Fig. 3. The pressure jump across the interface is obvious. However, the jump is not abrupt. In the CSF formulation, the surface tension force acts within a finite thickness of 2ε around the interface. The pressure varies smoothly across this interfacial thickness. Nevertheless, the magnitude of the pressure jump across the interface remains unaltered and is captured correctly. The pressure jump is larger in the developing region due to the larger curvature. The pressure jump reduces gradually as the curvature decreases along the flow direction.

Comparison is made with the pressure field predicted by VOF. There is good agreement between the two predictions except at x/W = 0.2. At this section, although both predict a similar trend, the pressure distribution is quantitatively different. The maximum error (pVOF − pLSET)/(pmax − pmin) with all values at x/W = 0.2, is around 6%.

The effect of surface tension on the interface evolution is shown in Fig. 4. Figure 4a shows that the interfaces predicted by the
present approach are in good agreement with that of the VOF (with \( \delta_{in}/W = 0.3 \)). Once fully developed flow is attained, the interface converges to the same location irrespective of \( Ca \). As the boundary layer develops in the inlet region, curvature of the interface is created. This nonzero interface curvature leads to a surface tension force that tends to flatten the interface. This, in turn, hinders the development of the boundary layer. Therefore, the entry length increases for smaller \( Ca \) flow where the surface tension force is larger.

For flow without surface tension, the fully developed velocity profiles and \( \delta_E/W \) are functions of \( \mu_1/\mu_2 \) and \( V_1/V_2 \) and independent of \( \delta_{in}/W \) (Refs. 5, 7, and 8). This is still valid for the case with surface tension because surface tension only affects the flow characteristics in the developing region. For demonstration, \( \delta_{in}/W \) is varied while fixing \( V_1/V_2 \) and \( \mu_1/\mu_2 \) to 2.0, respectively. The inlet velocities of both fluids have to be modified accordingly to maintain the same \( V_1/V_2 \) for different \( \delta_{in}/W \). Figure 4b shows the evolutions of the interface for \( \delta_{in}/W = 0.3, 0.5, \) and 0.7, respectively, with and without surface tension. Although the interface locations at the inlet are different, all of the fully developed interfaces converge to the same location irrespective of the presence of surface tension. Again, the entry length increases with the presence of surface tension.

IV. Conclusions

The effect of surface tension on a stratified two-phase flow between parallel plates is investigated using the LSET method with a recently proposed LMC scheme. The surface tension term is modeled using the CSF model. The results from the present Note agree well with the results obtained using the VOF method. It is found that surface tension effect is confined to the developing region where the interface curvature is nonzero. The entry length increases with the coefficient of surface tension. However, the interface location and velocity profile in the fully developed region are independent of surface tension.

References


